Maths Course Guide 2018 – 2019

Foundation Mathematics

A Guide

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Contents

Mathematics:

The course is created for students that are aiming to continue their studies at universities in science or business and economics related areas. It is a combination of Pure Maths and Applied Mathematics for Science or Business and Economics and is an equivalent of A-level.

The first section of the course is based on Pure Mathematics (such as numeracy, algebra, powers, logarithms, functions and graphs). It also includes learning of inequalities, algebraic division, basic differentiation and integration. The main topics in this section are Simultaneous Equations, Polynomials, Factorising and Sketching.

The second section of the course comprises Linear Programming, Exponential Function, also Financial Mathematics with Economic Function or Vectors. Students will learn how to minimise the Cost or maximise Revenue and Profit. They will also know how the banking system and exponential relations are based on mathematics of Arithmetic and Geometric Progressions. Students will develop their skills of solving Simultaneous Equations and Inequalities, both algebraically and graphically, and learn about Vectors, Exponential Growth and Decay in application with real life problems. They will see that the use of Powers and Logarithms they learned in the beginning is essential for solving financial and scientific tasks.

In the final section Humanities students will study Statistics, learn about different graphs, averages and measures of spread. They will study probability and some elements of mathematical modelling, while obtaining equation of Regression Line. The knowledge of Linear Function's Graph gained during the course of Pure Mathematics will prove essential for this section. Science Foundation students will develop their knowledge of Pure Mathematics, with the use of more complicated topics applied to solve sophisticated problems in Calculus, Coordinate Geometry and Transformations of Functions. This study will develop further their skills of making decisions and solving real life problems.

An important feature of the course is the use of computer generated graphical methods of presenting questions and solving problems. All Foundation students will learn how to use graphical and equation editing computer programs, to be able to produce excellent electronic version of their Research in Mathematics. Their knowledge and skills in using Microsoft Office or Apple Macintosh will also well develop during the second term of the course. Smart Board interactive and Workbook method of teaching is well established and presented.

A requirement to a participant will be an equivalent of GCSE, as well as some degree of interest in precise methods and calculation. Every student is invited to use their own laptops for their personal Research in Applied Mathematics of the second term. With the course of Foundation Mathematics, students will develop both their mental mathematical skills and the use of a scientific calculator. They will improve and gain new skills in the use of computer programs and software. The course will make graduates ready to solve problems, make decisions and apply their knowledge to real life. They will understand how widely mathematical methods are used in variety of real life situations, in the chosen area of their studies.

Humanities Foundation Mathematics 2018 – 2019

Term 1 - Pure Mathematics 15 weeks	Exam 1 – Pure Mathematics
Types of Numbers Ratios & Percentages Powers & Logarithms Algebra & Quadratics Functions & Graphs Inequalities Simultaneous Equations Differentiation & Integration Polynomials & Cubic Graphs	Please see a typical Exam problem in section 12
Term 2 - Applied Mathematics & Statistics 12 weeks	Research - Applied Mathematics
Economic Functions Linear Programming Sequences & Series (AP & GP) Financial Mathematics Exponential Growth & Decay Types of Data Types of Representation of Data Types of Distribution of Data Stem & Leaf Diagram	Please see a typical Research problem in section 12
Term 3 - Statistics: 7 weeks	Final Exam (2 Papers) – Applied Mathematics & Statistics
Histogram Cumulative Frequency Graph & Box Plot Averages Measures of Spread Criteria for Outliers Scatter Graph & Types of Correlation PMCC Equation of Regression Line	Please see typical Exam problems in section 12

Science-Biology Foundation 2018 – 2019

Term 1 - Pure Mathematics 15 weeks	Exam 1 – Core Mathematics 1
Types of Numbers Ratios & Percentages Powers & Logarithms Algebra & Quadratics Functions & Graphs Inequalities Simultaneous Equations Differentiation & Integration Polynomials & Cubic Graphs	Please see a typical Exam problem in section 12
Term 2 - Applied Mathematics & Core Maths 2 12 weeks	Research - Applied Mathematics
Sequences & Series (AP & GP) Exponential Function Exponential Growth & Decay Vectors & Modelling The Use of Differential Calculus The Use of Integral Calculus Equation of a Straight Line Equation of a Circle Calculus & Coordinate Geometry	Please see a typical Research problem in section 12
Term 3 - Statistics: 7 weeks	Final Exam (2 Papers) – Core 2 & Applied Mathematics
Integration as the Reverse of Differentiation Transformations of Functions Reciprocal Function Transformations of Reciprocal Function The Binomial Expansion Basic Trigonometry Trigonometry & Radian Measure Trigonometry in a Circle	Please see typical Exam problems in section 12

Mathematics

The mathematics programme has been reviewed to reflect and cater for the actual mathematical requirements of students as they progress towards their university courses. It's designed to lay the foundation for developing and consolidating effective reasoning and a methodical approach while building a good set of mathematical skills relevant to most science and humanities studies. The program aims to provide students with a valuable range of tools and techniques for analysing, modelling, formulating and solving general mathematical problems that can arise in their further studies or future practice.

Mathematics – Specification Summary

Term 1	**Term 2**	**Term3**
○ Basic Mathematics ○ Powers & Logarithms ○ Algebra & Quadratics ○ Functions and their Graphs ○ Equations & Inequalities ○ Differentiation & Integration ○ Polynomials, Factorising & Sketching	○ Economic Functions ○ Linear programming ○ Sequences and Series ○ Financial Mathematics ○ Exponential Growth & Decay ○ Representation of Data ○ Averages & Measures of Spread	○ Probability ○ Scatter Graphs & Correlation ○ PMCC & Regression Line ○ Coordinate Geometry ○ The Use of Calculus & Trapezium Rule ○ Transformations of Functions ○ Reciprocal Function ○ Trigonometry & Radian Measure

NB: To keep the same standard for assessment purpose, effort should be made to cover the material for each term in the term indicated. However, within each term the content may be covered in any suitable order and some components may be exceptionally moved from one term or another to respond to the students' level of attainment or if required for use by other subjects.

Pre-requisite

An intermediate level in mathematics is suitable, equivalent to GCSE foundation standard.

Assessment

All students will go through assessments as part of their course as shown below:

1. All students will undertake an exam at the beginning of their course to attain a starting level so that progress can be measured.
2. January each year, one exam based on content covered to date. This assessment will be used to obtain an understanding of the ability of the student and to acquire knowledge of the progression of the student based on test one above. There will be one exam for all abilities.
3. All students will carry out a research paper (40% of the total mark) based on a brief given to them by their teachers. The research papers will need to be submitted by the end of the Easter term each year. One brief with a variety of tasks will be given to all students. The research brief will differ depending on the course being studied; science or humanities.

4. Final exams will take place in May & June (60% of the total mark) each year where students will sit two papers. The papers will cover the work carried out during the academic year. There will be two exams for all abilities.

Specification content

Basic Mathematics

What students need to learn:

- Types of number : natural, integer, decimal, rational, irrational and real numbers
- The methods of rounding numbers : 'to the nearest', to certain number of decimal places or significant figures
- Working with ratios and percentages to express or find shares from a whole quantity

Powers & Logarithms

What students need to learn:

- Working with particular forms of number such as reciprocals, indices (or powers), fractions and surds. Students should in particular learn the properties and know how to work with fractions, indices and surds including how to rationalise the denominator
- Logarithm as the expression of power
- Logarithm laws including the formula for changing the base
- Solving various exponential and logarithmic equations

Algebra & Quadratics

What students need to learn:

- Algebraic expressions and related operations including collecting like terms, multiplying out the brackets, simplifying algebraic expressions and solving linear equations
- Basic quadratic function and its graph and the use of the discriminant, factorisation and completing the square method for sketching the graphs of quadratic functions of the form $y = ax^2 + bx + c$ and solving quadratic equations by using different methods, including the quadratic formula

Functions & their Graphs

What students need to learn:

- Basic linear function of the equation $y = x$ and its graph, slope (gradient) and the intercept of a linear function of the form $y = mx + c$
- Relate the equations of the function with their graphs, recognise the shape of the graphs by their equation
- Rules for sketching linear graphs and the method of using the intercept and zero-point for drawing straight line graphs
- Sketch the graphs of circles using the equation $(x – a)^2 + (y – b)^2 = r^2$

Equations & Inequalities

What students need to learn:

- Equations: differentiating between, expressions, equations, identities and functions. Solving linear, quadratic and simple cubic equations. Solving simultaneous linear equations using elimination, substitution or equating as appropriate. Solving simultaneous mixed equations (linear and non-linear) and presenting the solutions in a suitable form
- Solving linear, quadratic and simultaneous inequalities. For quadratic inequalities, the curve can be used along the sign inspection methods

Differentiation

What students need to learn:

- The concept of differentiation and its relation to the gradient of the tangent to a curve
- Basic rules of differentiation for polynomials and algebraic functions with rational Indices
- Second and third derivatives
- Use of differentiation to determine the set of values for which a differentiable function is increasing or decreasing
- Use of differentiation to find Stationary points and determine their nature
- Sketch the graph of elementary Quartic functions

Integration

What students need to learn:

- Indefinite integration as the reverse process of differentiation
- Basic rules of integration for polynomials and algebraic functions with rational Indices
- Definite integral and its meaning as the area under a curve
- Calculation of definite integrals using given limits

Polynomials, Factorising & Sketching

What students need to learn:

- Algebraic expressions and related operations including determining the degree and coefficients of a polynomial, addition, subtraction, multiplication, simplification, expansion, and factorisation
- Algebraic fractions and related operations including simplification, long division by a linear term, the remainder theorem and the factor theorem
- Sketch the graphs of linear, quadratic and cubic functions by using general guidance for sketching curves with finding the points of intersection with the axes, stationary points by differentiation and the limits of the function

Economic Functions

What students need to learn:

- Apply the concepts of linear and quadratic functions for business and economics on the examples of Demand, Supply, Cost and Total Revenue and Profit functions and their graphs
- Use simultaneous equations method for solving problems with Equilibrium Point and Break Even Points, both algebraically and graphically
- Use differentiation for finding Marginal functions and Maximum Revenue or Maximum Profit

- Solve various problems with Economic functions and their graphs

Linear Programming

What students need to learn:

- Standard form of a linear problem: the variables, the constraints and the objective function
- Modelling a variety of problems using linear programming: examples can be drawn from business, transport, manufacturing and other sectors
- Graphical representation of the feasible region
- Finding a solution graphically using the objective-line method
- Finding a solution using the vertex inspection method

Sequences and Series

What students need to learn:

- General concepts of a sequence and series: 1st term, general term, recurrence relation, sum of first n terms, the use of Sigma notation
- Arithmetic sequence and series
- Geometric sequences and series including sum to infinity where defined
- Solving real life problems involving sequences and series

Financial Mathematics

What students need to learn:

- Simple interest
- Compound interest: interest compounded annually, semi-annually, monthly or n times per year on regular intervals
- Continuously compounded interest
- Future and Present values and Annual Percentage Rate
- Debt Repayment and Annuities

Exponential Growth & Decay

What students need to learn:

- The function a^x and its graph and properties
- Graph of logarithm function with base a
- Solving logarithm and exponential equations and simple inequalities
- Use of exponential and logarithm functions to model growth and decay in a population or for solving various scientific problems

Representation of Data

What students need to learn:

- Types of data, qualitative, quantitative, discrete and continuous data
- Types of representation of data: Stem and Leaf diagram, Bar and Pie Charts, various graphs
- Frequency Density and Histograms
- Cumulative frequency graph and Box & Whisker Plot

Averages & Measures of Spread

What students need to learn:

- Analyses and summary (for both discrete and continuous data): the use of frequency, cumulative frequency, mode, median and quartiles, inter-quartile range, mean and standard deviation
- Solve real life problems involving comparing standard deviation and overall mean and standard deviation
- Criteria for Outliers and their use for solving real life problems

Probability

What students need to learn:

- Probability concepts and probability tools: trial, outcome, sample space, event, complementary event, compound events, mutually exclusive events, independent events
- Representation of events using multi-dimensional tables, Venn diagram and tree diagrams
- Probability Laws
- Unconditional and Conditional probability

Scatter Graph & Types of Correlation

What students need to learn:

- Bivariate data: scatter diagrams and Line of best fit
- Types of correlation and different methods of measuring correlation: comparing variables, line of best fit, ellipse & circle method
- Comparing measures of correlation

PMCC & Regression Line

What students need to learn:

- Product moment correlation coefficient as a numerical measure of correlation
- Solving real life problems by using PMCC and its interpretation
- Explanatory and response variable and linear regression
- Solve real life problems with Equation of regression line and its graph
- Prediction and estimation from regression model

Coordinate Geometry

What students need to learn:

- Cartesian equation of a straight line in a system of axes in different forms such as $y = mx + c$, $ax + by + c = 0$ or $y - y_1 = m(x - x_1)$
- Parallel and perpendicular straight lines, intersection of 2 or more straight lines
- Coordinates of the midpoint of a segment and distance between two points
- Cartesian equation of a circle in a system of axes in different forms such as $(x - a)^2 + (y - b)^2 = r^2$ and $x^2 + y^2 + px + qy + r = 0$
- Circle properties and their use in solving problems
- Solving general problems involving straight lines, circles and other common shapes

The Use of Calculus

What students need to learn:

- Equation of the tangent and equation of the normal at a given point on the curve y = f(X)
- General problems involving differentiation and coordinate geometry
- Use differentiation to solve simple optimisation problems
- Finding the constant of integration given the initial conditions
- Area under a curve, area between a curve and a straight line
- The Trapezium rule as a method of numerical integration

Transformations of Functions

What students need to learn:

- Linear function as the transformation of the basic linear function y = x
- Quadratic function as the transformation of the basic quadratic function $y = x^2$
- Cubic function as the transformation of the basic cubic function $y = x^3$
- Transformation of curves: y = f(x + a), y = f(x) + a, y = f(ax), y = af(x), y = -f(x) and y = f(-x). Students should be able to correctly describe each transformation and apply it to sketch the corresponding curve based on the curve y = f(x)
- Sketching graphs of simple functions including linear, quadratic and cubic by transformation

Reciprocal Function

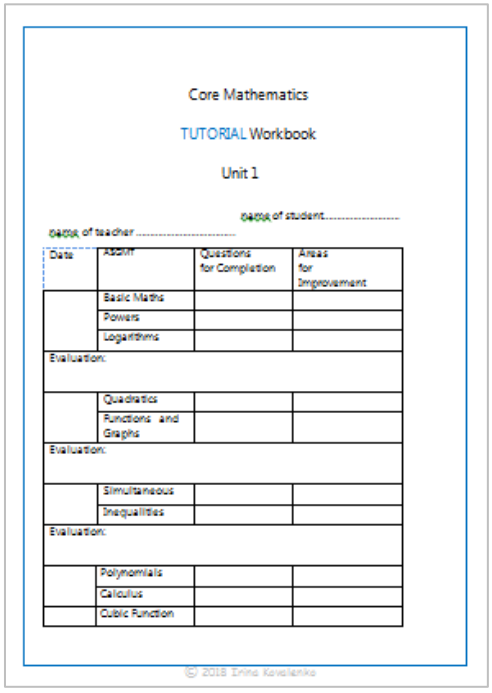
What students need to learn:

- Sketching graphs of simple Reciprocal functions ($y = \frac{a}{x}$). The concepts of limits and continuity are not in the scope of this specification, but the vertical or horizontal asymptotes and infinite branches must be determined and used where required
- Apply transformations for sketching various reciprocal graphs with indication of the asymptotes

Trigonometry & Radian Measure

What students need to learn:

- Basic trigonometric ratios and identities, Sine and Cosine rules
- Area of a triangle using Sine of the angle between its sides
- Radian as an alternative measure of angle, conversions between Radians and Degrees
- The length of an ark, area of a sector and segment using Radians
- Solve geometric problems with the use of radians, arcs, sectors and segments
- Unit circle, quadrants, trigonometric ratios and graphs of trigonometric functions
- The use of Unit circle for solving linear and quadratic trigonometric equations



Core Mathematics

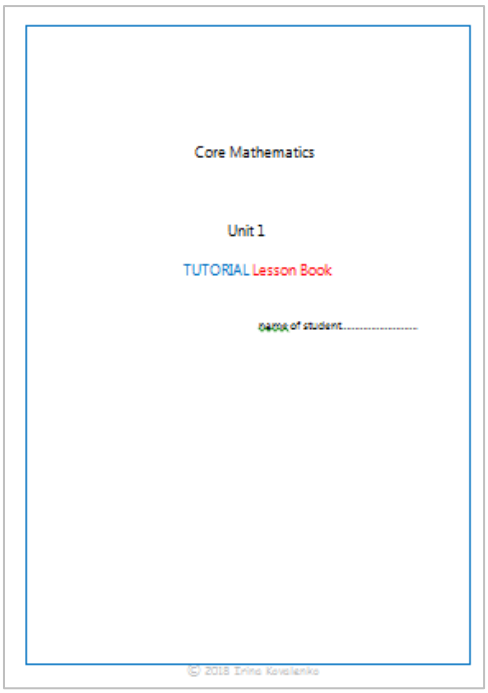
TUTORIAL Workbook

Unit 1

Name of student.....

Name of teacher .....

Date	Activity	Questions for Completion	Areas for Improvement
	Basic Maths		
	Powers		
	Logarithms		
Evaluation:			
	Quadratics		
	Functions and Graphs		
Evaluation:			
	Simultaneous Inequalities		
Evaluation:			
	Polynomials		
	Calculus		
	Cubic Function		



Core Mathematics

Unit 1

TUTORIAL Lesson Book

Name of student.....

Tutorial Books are made for working in lessons (Tutorial Workbook) and at home, if a lesson or two had been missed for an authorised reason (Tutorial Lesson Book).

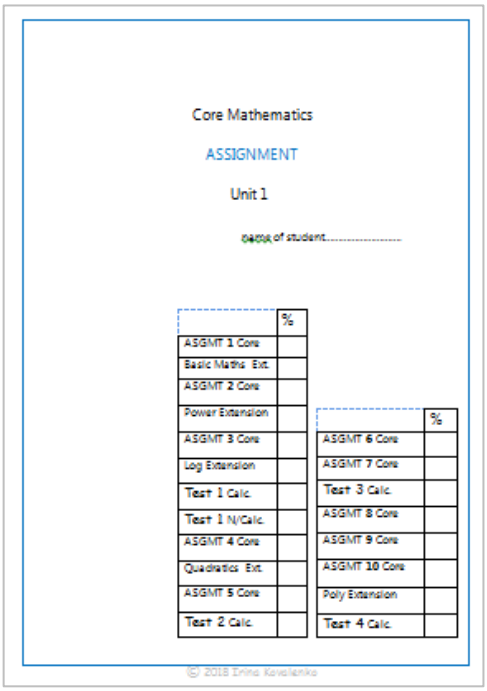
There are 4 Units, covering the course, with 4 different colours of the page boarders. For example, Unit 1 has a Blue border on the front page.

Each student gets a Tutorial Workbook to follow lessons: write down definitions, solve problems and draw graphs - among other students in the classroom.

Tutorial Lesson Book contains most of the definitions, solutions and graphs. It can be given to a student by the teacher, if necessary.

All students are expected to keep their Tutorial Books in excellent order, bring them to every lesson and work in them with good effort during the class.

Tutorial Books are being checked and signed by the teacher a few times during each term, usually at the time of regular term tests.


Core Mathematics

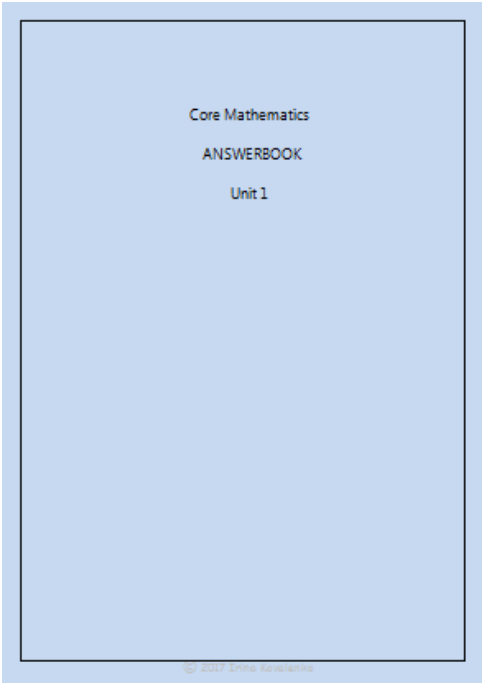
ASSIGNMENT

Unit 1

name of student.....

	%
ASGMT 1 Core	
Basic Maths Ext.	
ASGMT 2 Core	
Power Extension	
ASGMT 3 Core	
Log Extension	
Test 1 Calc.	
Test 1 N/Calc.	
ASGMT 4 Core	
Quadratics Ext.	
ASGMT 5 Core	
Test 2 Calc.	

	%
ASGMT 6 Core	
ASGMT 7 Core	
Test 3 Calc.	
ASGMT 8 Core	
ASGMT 9 Core	
ASGMT 10 Core	
Poly Extension	
Test 4 Calc.	


Core Mathematics

ANSWERBOOK

Unit 1

Assignment Books are made for working individually at home or in lesson. They are students' main exercise books and the best help in preparing for tests, research and exams.

There are 4 Units, covering the course, with 4 different colours of the page boarders. For example, Unit 1 has a Blue border on the front page.

Each student gets an Assignment Book for their practice. Answer Books are available in electronic form and usually shown on a Smart Board in the beginning of each lesson during homework check. The answers to each Assignment are checked only once and usually are not checked again.

There are also a few paper versions of the Answer Books for an exceptional use.

All students are expected to keep their Assignment Books in excellent order, bring them to every lesson and work in them with good effort at home or in class. Every lesson students are given homework and the deadline for its completion. These deadlines should be strictly followed.

Assignment Books are being checked and signed by the teacher a few times during each term, usually at the time of regular term tests.

Vocabulary

Vocabulary is given at the end of each Tutorial Book and sorted by topics.

A sample of topics' vocabulary is given below.

Students are expected to learn their vocabulary before the beginning of every new topic.

ALGEBRA and QUADRATICS

Expression

– a combination of letters, numbers and signs in mathematics.

Example: $2x+3$ or $2x^2+3xy+5y^2$.

Constant

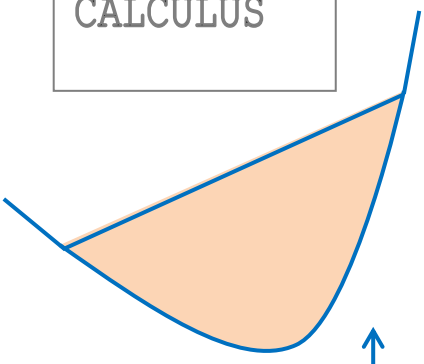
– a fixed value, a number on its own; opposite to the variable (x).

Example: $2x^2+3x+5$.

CALCULUS

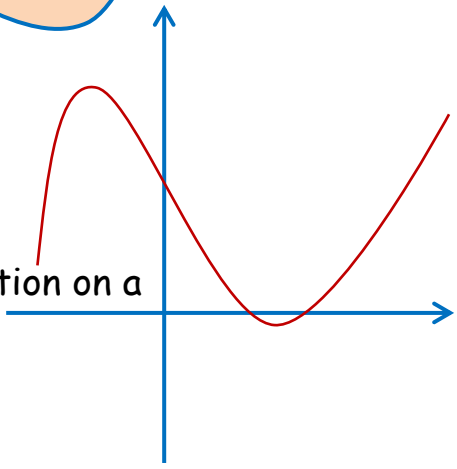
Area

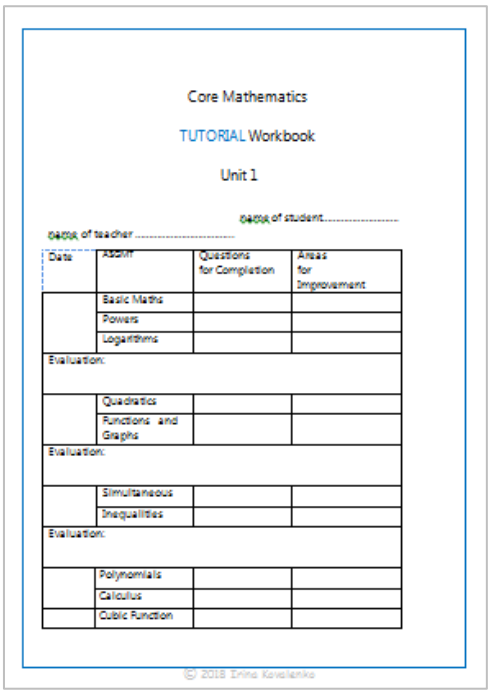
– the amount of space inside a shape.



Curve

– a bending line, without angles / the graph of a function on a coordinate plane.





Core Mathematics

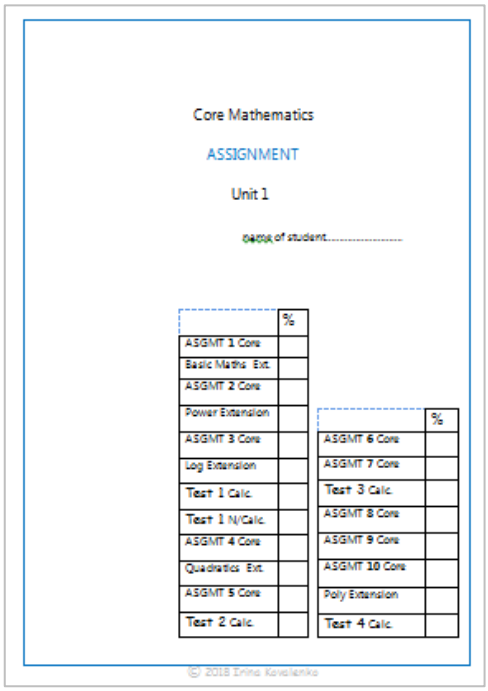
TUTORIAL Workbook

Unit 1

Name of student.....

Name of teacher .....

Date	ASGMT	Questions for Completion	Areas for Improvement
	Basic Maths		
	Powers		
	Logarithms		
Evaluation:			
	Quadratics		
	Functions and Graphs		
Evaluation:			
	Simultaneous		
	Inequalities		
Evaluation:			
	Polynomials		
	Calculus		
	Cubic Function		



Core Mathematics

ASSIGNMENT

Unit 1

Name of student.....

	%
ASGMT 1 Core	
Basic Maths Ext.	
ASGMT 2 Core	
Power Extension	
ASGMT 3 Core	
Log Extension	
Test 1 Calc.	
Test 1 N/Calc.	
ASGMT 4 Core	
Quadratics Ext.	
ASGMT 5 Core	
Test 2 Calc.	

	%
ASGMT 6 Core	
ASGMT 7 Core	
Test 3 Calc.	
ASGMT 8 Core	
ASGMT 9 Core	
ASGMT 10 Core	
Poly Extension	
Test 4 Calc.	

Unit 1 – Core Mathematics. It gives students the main concepts and definitions needed for the course. Also developes the skills of numeracy, algebra, solving equation and inequalities, sketching graphs and finding intersections.

Typical Vocabulary

Power/Index/Exponent

- a small number written to the right and above the base. Example: $8^2 = 8 \times 8 = 64$.

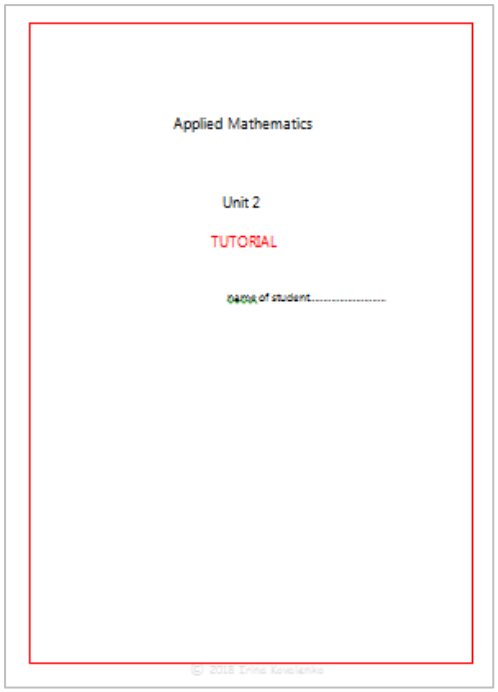
Base

Factorising

- breaking up into factors that can be multiplied together to get the original expression. Example: $x^2 - 2x - 8 = (x-4)(x+2)$.

Discriminant

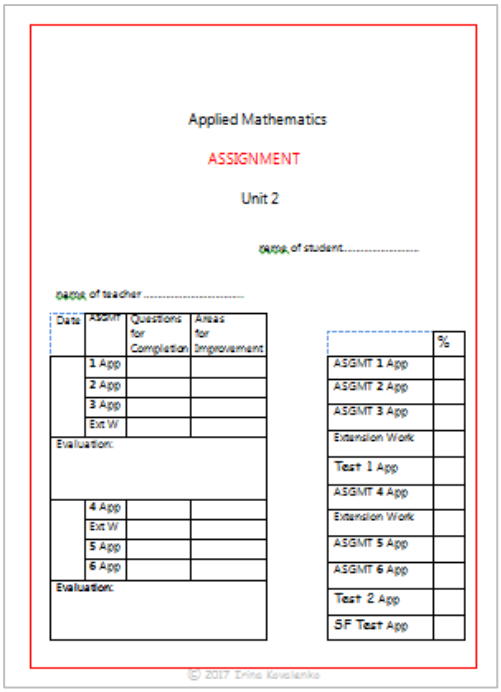
- an expression that appears under the square root sign in the quadratic formula: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.


Applied Mathematics

Unit 2

TUTORIAL

name of student.....


Applied Mathematics

ASSIGNMENT

Unit 2

name of student.....

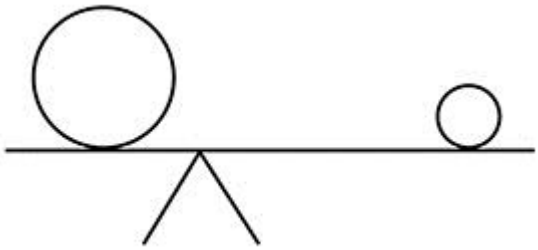
name of teacher .....

Date	ASGMT	Questions for Completion	Areas for Improvement
	1 App		
	2 App		
	3 App		
	Ext W		
Evaluation:			
	4 App		
	Ext W		
	5 App		
	6 App		
Evaluation:			

	%
ASGMT 1 App	
ASGMT 2 App	
ASGMT 3 App	
Extension Work	
Test 1 App	
ASGMT 4 App	
Extension Work	
ASGMT 5 App	
ASGMT 6 App	
Test 2 App	
SF Test App	

Unit 2 – Applied Mathematics. It teaches how to use Mathematics to solve real life problems. It is based on the use of Unit 1 and helps students to handle their Research in the second term.

Typical Vocabulary

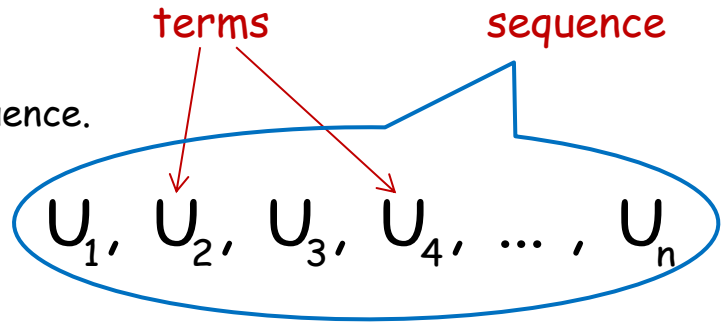


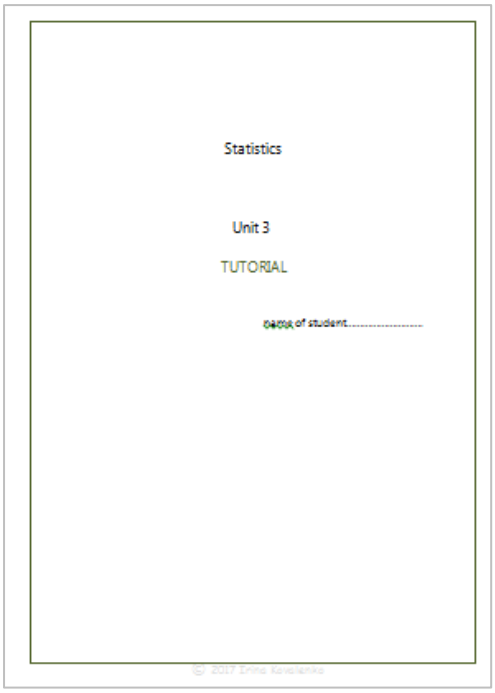
Equilibrium

- a state of balance between opposing forces or actions, for example between market supply and demand.

Term

- each singular value in a sequence.

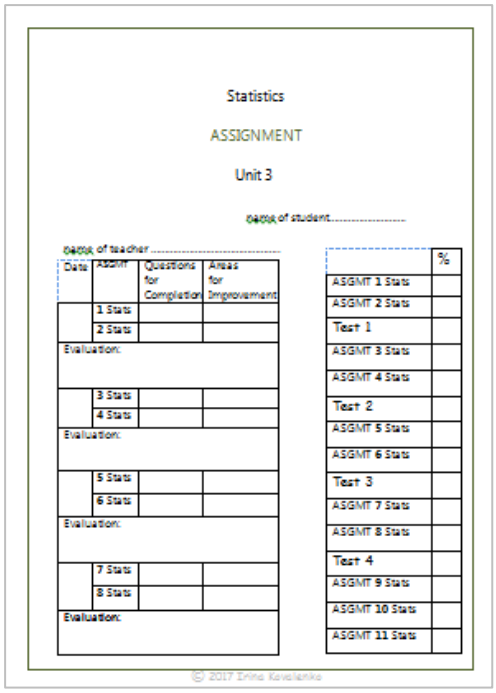



Statistics

Unit 3

TUTORIAL

Name of student.....


Statistics

ASSIGNMENT

Unit 3

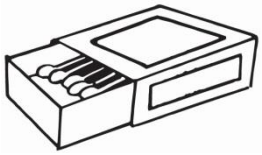
Name of student.....

Name of teacher .....

Date	ASSMT	Questions for Completion	Areas for Improvement
	1 Stats		
	2 Stats		
Evaluation:			
	3 Stats		
	4 Stats		
Evaluation:			
	5 Stats		
	6 Stats		
Evaluation:			
	7 Stats		
	8 Stats		
Evaluation:			

	%
ASSMT 1 Stats	
ASSMT 2 Stats	
Test 1	
ASSMT 3 Stats	
ASSMT 4 Stats	
Test 2	
ASSMT 5 Stats	
ASSMT 6 Stats	
Test 3	
ASSMT 7 Stats	
ASSMT 8 Stats	
Test 4	
ASSMT 9 Stats	
ASSMT 10 Stats	
ASSMT 11 Stats	

Unit 3 – Statistics. It is also a part of Applied Mathematics and serves the purpose of solving real life problems. It partly applies the knowledge of Unit 1 developing the skills of drawing graphs and using mathematical models.



Typical Vocabulary

Frequency

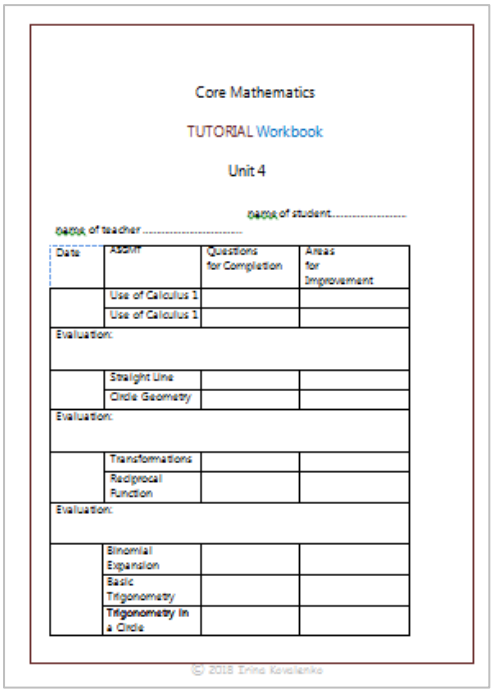
– the number of times the event occurred in an experiment or study. Example: number of times the number of matches in a matchbox being 39 (f = 11) or number of people working in a factory (f = 120) .

Mean Average

– the value obtained by dividing the sum of a set of quantities by the number of quantities in the set. It can be calculated by the formula:

$$\bar{x} = \frac{\sum xf}{\sum f}$$

, where x is quantitative data and f is frequency.



Core Mathematics

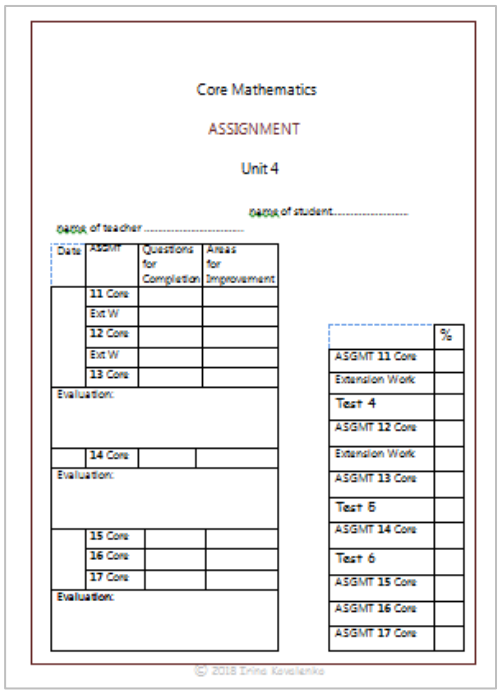
TUTORIAL Workbook

Unit 4

Name of student.....

Name of teacher .....

Date	Assmt	Questions for Completion	Areas for Improvement
	Use of Calculus 1		
	Use of Calculus 1		
Evaluation:			
	Straight Line		
	Circle Geometry		
Evaluation:			
	Transformations		
	Reciprocal Function		
Evaluation:			
	Binomial Expansion		
	Basic Trigonometry		
	Trigonometry in a Circle		



Core Mathematics

ASSIGNMENT

Unit 4

Name of student.....

Name of teacher .....

Date	Assmt	Questions for Completion	Areas for Improvement
	11 Core		
	Ext W		
	12 Core		
	Ext W		
	13 Core		
Evaluation:			
	14 Core		
Evaluation:			
	15 Core		
	16 Core		
	17 Core		
Evaluation:			

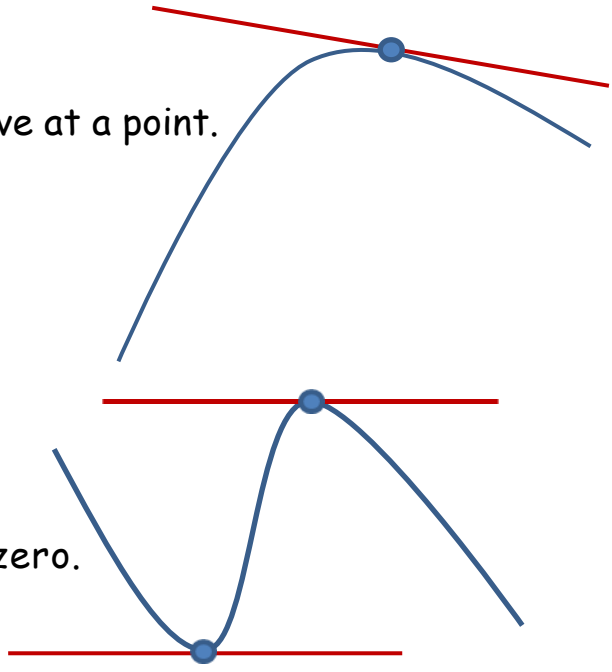
	%
ASGMT 11 Core	
Extension Work	
Test 4	
ASGMT 12 Core	
Extension Work	
ASGMT 13 Core	
Test 5	
ASGMT 14 Core	
Test 6	
ASGMT 15 Core	
ASGMT 16 Core	
ASGMT 17 Core	

Unit 4 – Pure Mathematics. This unit develops the concepts and methods of Unit 1, also brings in more topics and ideas. With this unit students have to take more creative approach in solving problems. Ideas and methods of Unit 4 demand students' deep understanding and flexibility at using Unit 1.

Typical Vocabulary

Tangent

- a straight line that "just touches" the curve at a point.



Stationary Point

- any point on a curve where gradient is zero.

Q1 Term1 Exam Pure Mathematics

Solve giving your answer to 3 significant figures:

$\log_2 x = 8 + 9\log_x 2$ **(7)**

$\log_4 x + 2\log_x 4 + 3 = 0$ **(7)**

(Total **14** marks)

Leave blank

Q1

Q1

Solve giving your answer to 3 significant figures: $\log_2 x = 8 + 9\log_x 2$ **(7)**

$\log_2 x = 8 + 9\dfrac{\log_2 2}{\log_2 x}$ $\quad (\log_2 x)^2 = 8\log_2 x + 9$ $\quad$ let $\log_2 x = y$

- $y^2 - 8y - 9 = 0$ $\quad (y-9)(y+1) = 0$ $\quad y_1 = 9$ $\quad y_2 = -1$ $\quad \log_2 x = 9$ or $\log_2 x = -1$
- $x_1 = 2^9 = 512$ $\quad x_2 = 2^{-1} = \dfrac{1}{2} = 0.50$

$\log_4 x + 2\log_x 4 + 3 = 0$ **(7)**

$\log_4 x + \dfrac{2}{\log_4 x} + 3 = 0$ $\quad$ let $\log_4 x = y$

- $y^2 + 3y + 2 = 0$ $\quad (y+2)(y+1) = 0$ $\quad y_1 = -2$ $\quad y_2 = -1$

  $\log_4 x = -2$ or $\log_4 x = -1$
- $x_1 = 4^{-2} = \dfrac{1}{16} = 0.0625$ $\quad x_2 = 4^{-1} = \dfrac{1}{4} = 0.25$

(Total **14** marks)

Q1

Q2 Term 2 Research Applied Mathematics

Leave blank

A bacterial culture starts with 500 bacteria and after 3 hours there are 8000 bacteria (assume exponential growth model).

a) Find an expression for the number of bacteria after t hours. **(3)**

$$P = P_0 e^{kt}$$

When $t = 0$, $P = P_0 = 500$

When $t = 3$, $500e^{k\times 3} = 8000$

$$e^{k\times 3} = \frac{80}{5} \qquad k\times 3 = \ln 16 \qquad k = \frac{\ln 16}{3}$$

$$\boxed{P = 500e^{\frac{\ln 16}{3}\times t}}$$

b) Find the number of bacteria after 4 hours. **(2)**

$t = 4$ $\qquad P = 500e^{\frac{\ln 16}{3}\times 4} = 20158.7368... \approx 20159$

c) When will the population reach 30000? **(4)**

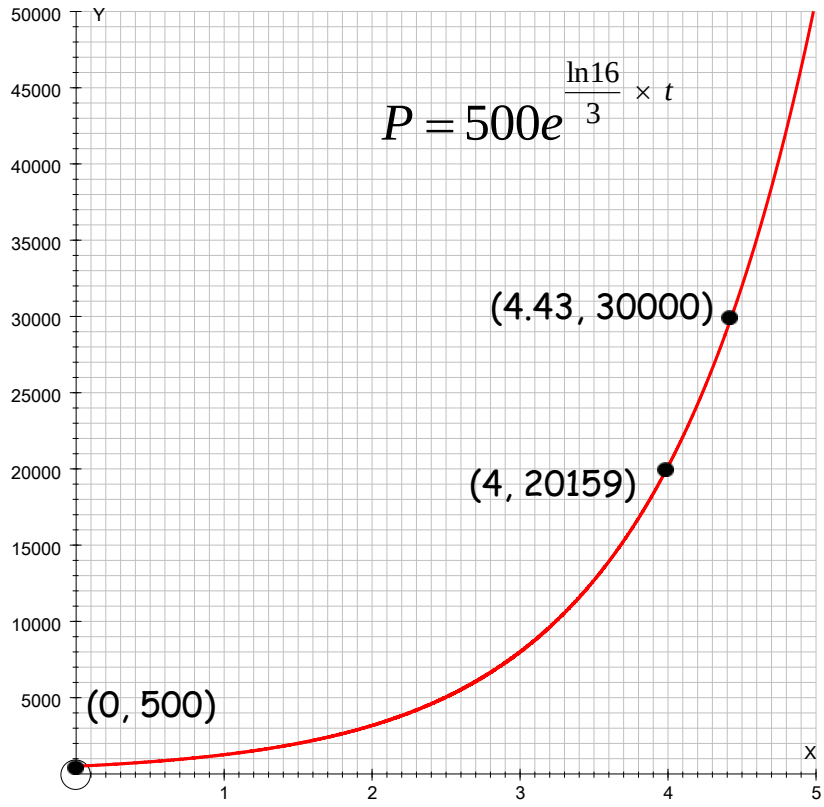
$$500e^{\frac{\ln 16}{3}\times t} = 30000$$

$$e^{\frac{\ln 16}{3}\times t} = 60 \qquad \frac{\ln 16}{3}\times t = \ln 60$$

Leave blank

$$t = \frac{3\ln 60}{\ln 16} = 4.430167947... \approx 4.43 \text{ hours}$$

d) Plot the expression from part (a) **(4)**



e) from the Plot in (d) what happens to the population as time increases? **(2)**

As time increases the population goes to infinity. The growth is incredibly fast, as already after 5 years the number of bacteria reaches (or exceeds) 50000.

Q2

(Total **15** marks)

Q3 Term 3 Exam Applied Mathematics

Leave blank

Given the demand function $Q = 150 - 0.5P$ and total cost function $TC = 564 + 14Q$.

a) Write down the equations for TR and the profit functions. **(3)**

the demand function $0.5P = 150 - Q$ $P = 300 - 2Q$

$TR = Q \times P$ $TR = Q \times (300 - 2Q)$ $\mathbf{TR = 300Q - 2Q^2}$

$\pi = TR - TC$

$\pi = 300Q - 2Q^2 - 564 - 14Q$

$\boldsymbol{\pi = -2Q^2 + 286Q - 564}$

b) Calculate the break-even points algebraically. **(2)**

$TR = TC$ $-2Q^2 + 300Q = 564 + 14Q$

$2Q^2 - 286Q + 564 = 0$

$Q^2 - 143Q + 282 = 0$

$(Q - 2) \times (Q - 141) = 0$

$Q_1 = 2$ **or** $Q_2 = 141$

$TR_1 = TC_1 = 564 + 28 = 592$

$TR_2 = TC_2 = 564 + 1974 = 2538$

c) Calculate the number of units which must be produced to maximize TR and the profit. **(3)**

$$\frac{dTR}{dQ} = -4Q + 300 \qquad \frac{d\pi}{dQ} = -4Q + 286$$

$-4Q + 300 = 0$ $-4Q + 286 = 0$

$Q = 75$ $Q = 71.5$

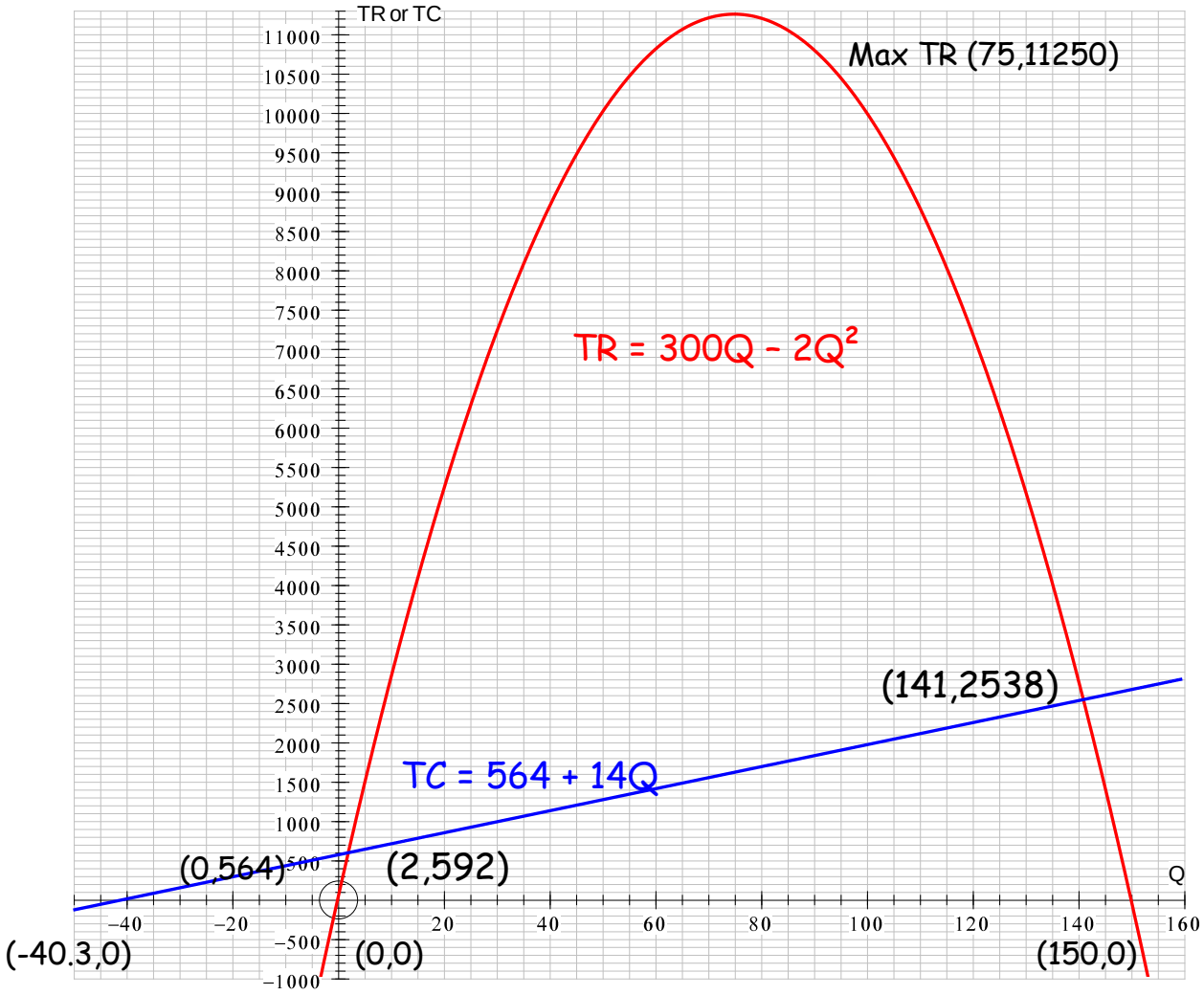
Leave blank

d) Using the grid below sketch the graphs of *TR* and *TC* functions. Show maximum *TR* and break-even points on the graph. (**6**)

TR (Q = 75) = 11250 (75, 11250)

TR = 0 Q × (300 – 2Q) = 0 Q = 0 **or** Q = 150
(0,0) (150,0)

TC = 564 + 14Q (0,564) (-40.3,0)



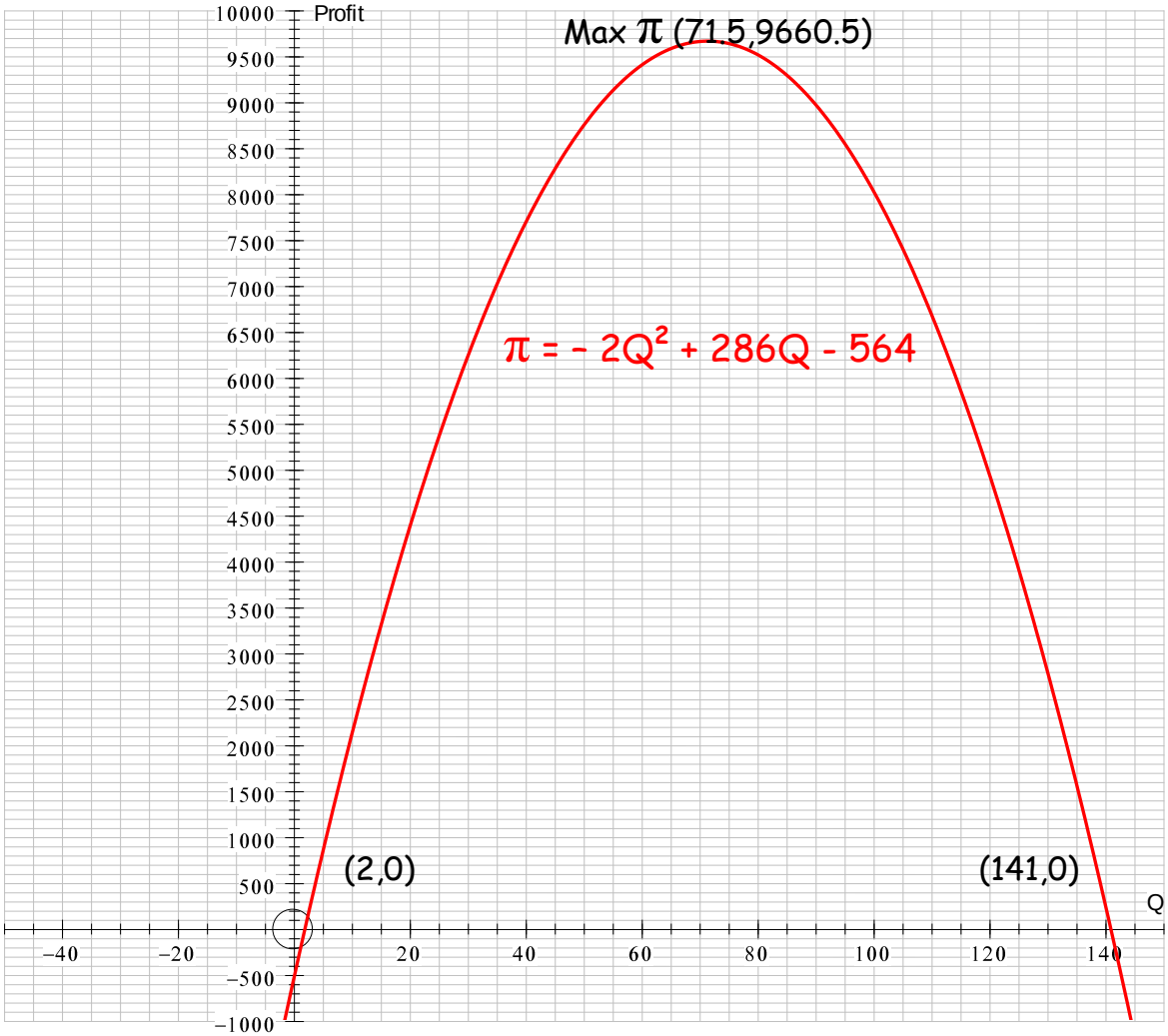
e) Using the grid below sketch the graph of the profit function. Show maximum profit and break-even points on the graph. **(3)**

Leave blank

π (Q = 71.5) = 9660.5 **(71.5, 9660.5)**

π = 0 (TR = TC) when Q = 2 **or** Q = 141 (break-even points)

(2,0) (141,0)

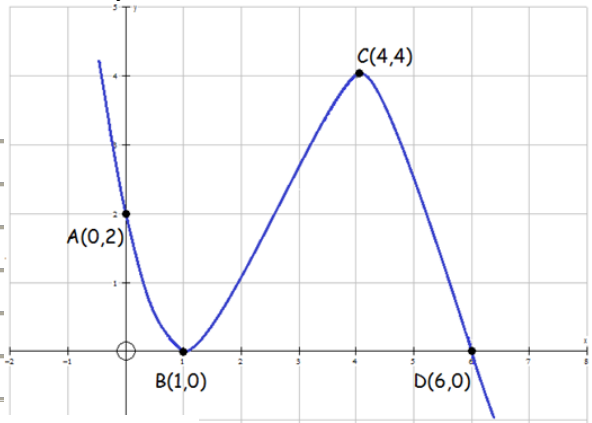


Q3

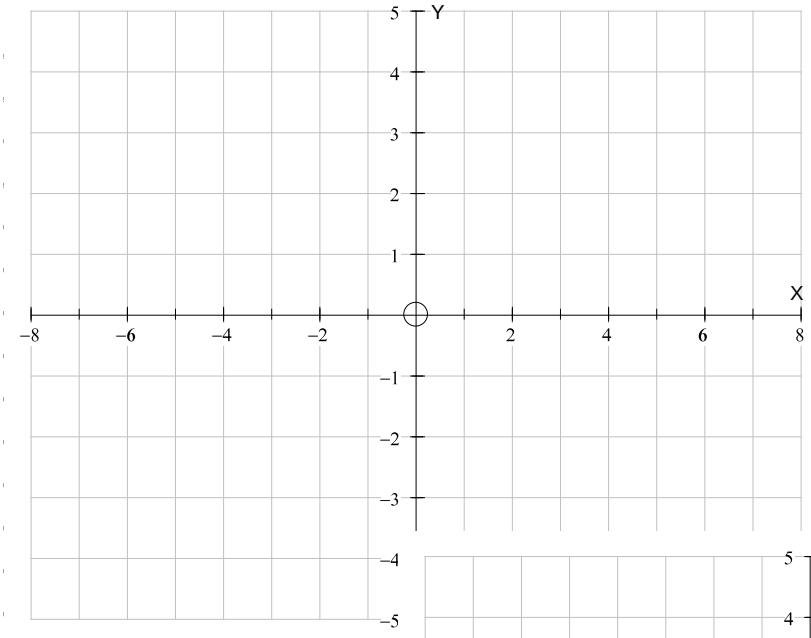
(Total 17 marks)

Q4 Term 3 Exam Pure Mathematics

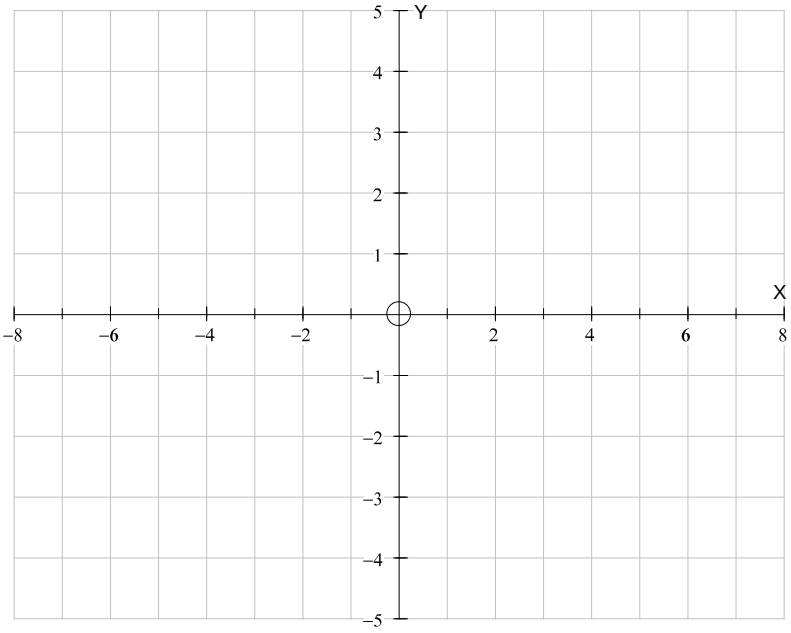
The following diagram shows a sketch of the curve with equation $y = f(x)$. The points $A(0, 2)$, $B(1, 0)$, $C(4, 4)$ and $D(6, 0)$ lie on the curve. Sketch the following graphs and give the coordinates of the points A, B, C and D after each transformation.



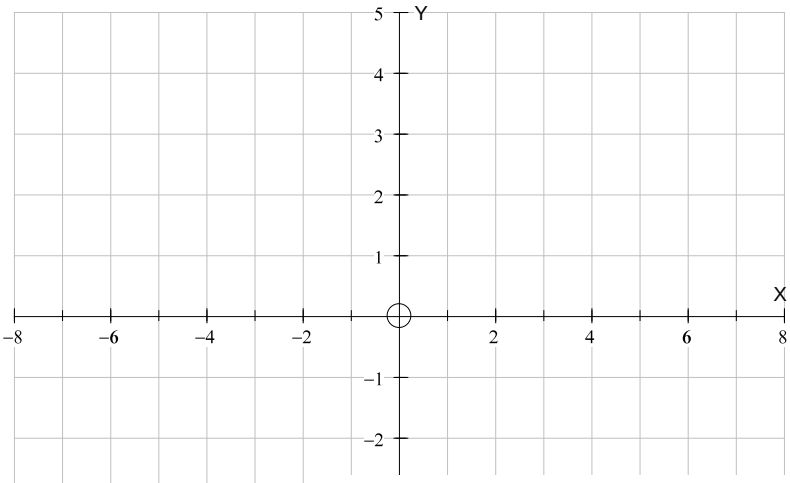
a) $f(x + 4)$ **(3)**



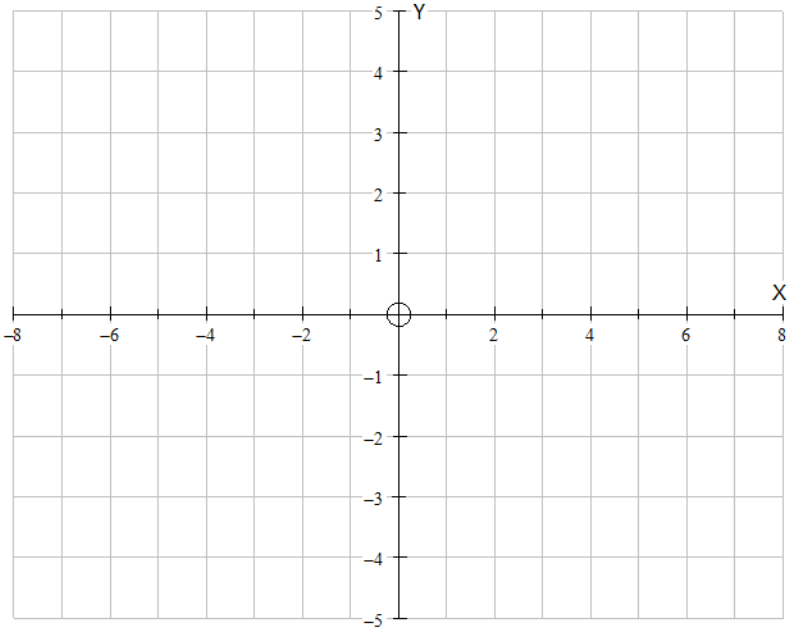
b) $f(x) - 2$ **(3)**



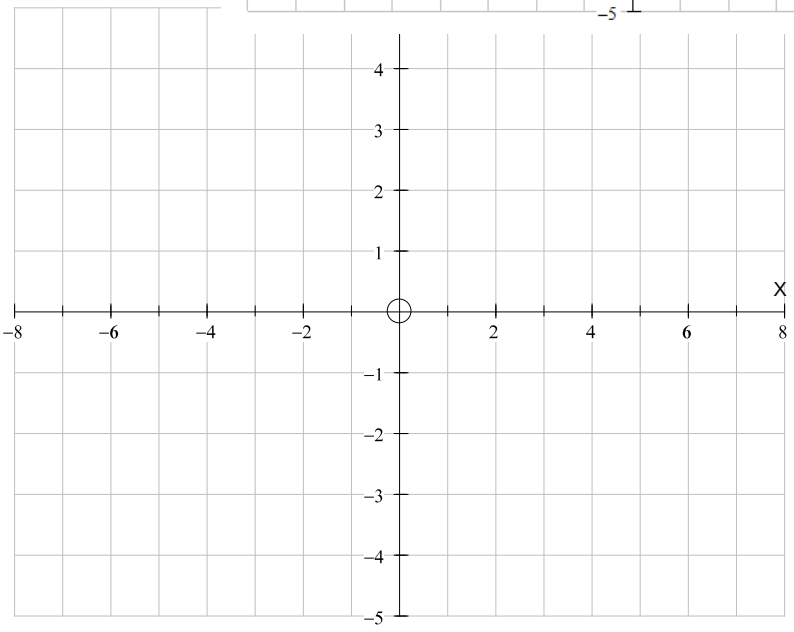
c) f(4x) **(3)**



d) − f(x) **(3)**



e) f(− x) **(3)**

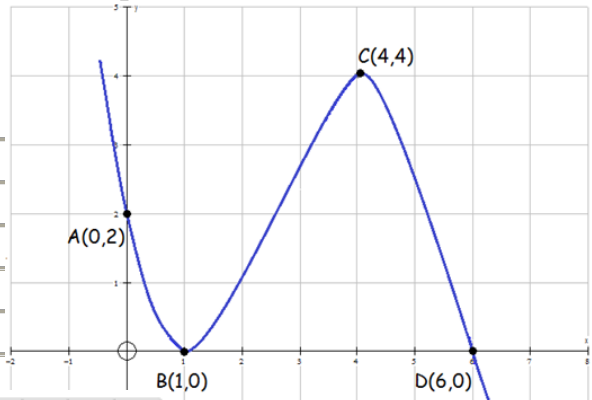


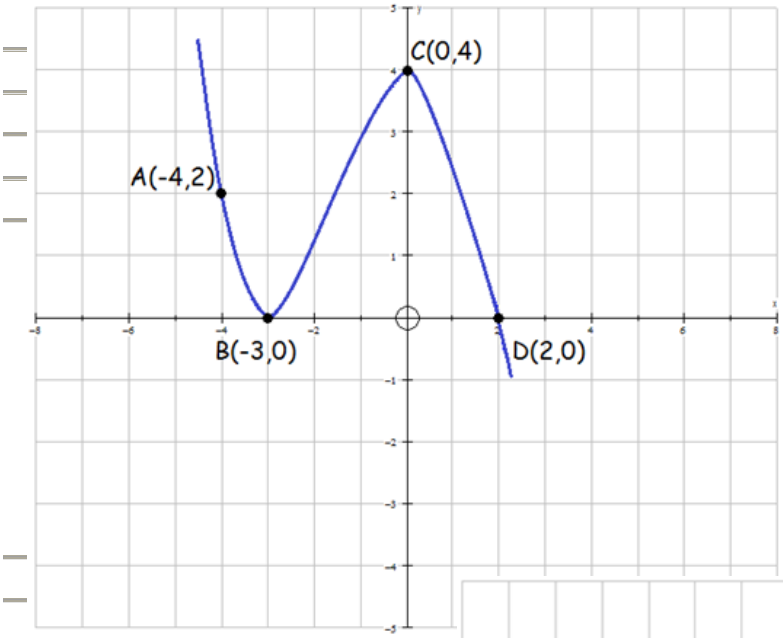
(Total **15** marks)

Q3

The following diagram shows a sketch of the curve with equation $y = f(x)$. The points $A(0, 2)$, $B(1, 0)$, $C(4, 4)$ and $D(6, 0)$ lie on the curve. Sketch the following graphs and give the coordinates of the points A, B, C and D after each transformation.

f) $f(x + 4)$ **(3)**

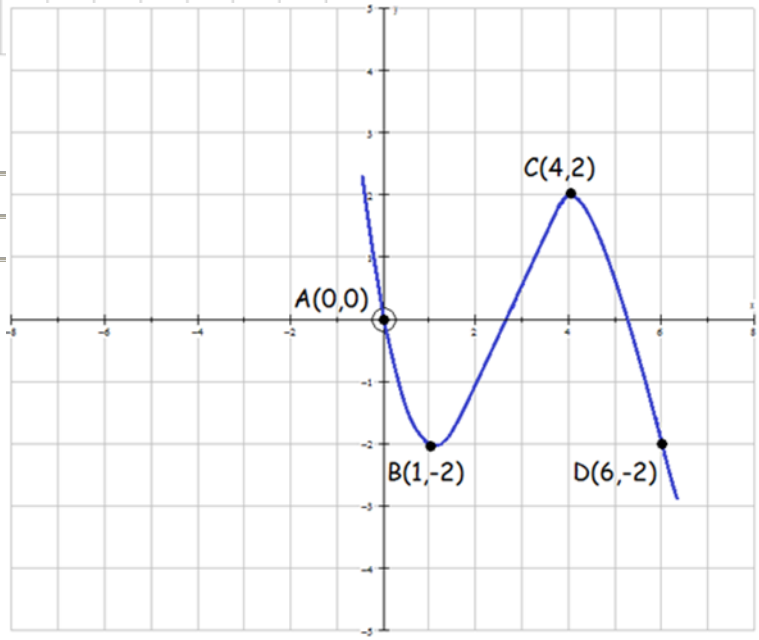




translation along OX
in a negative direction

moves every point to the left by 4 units

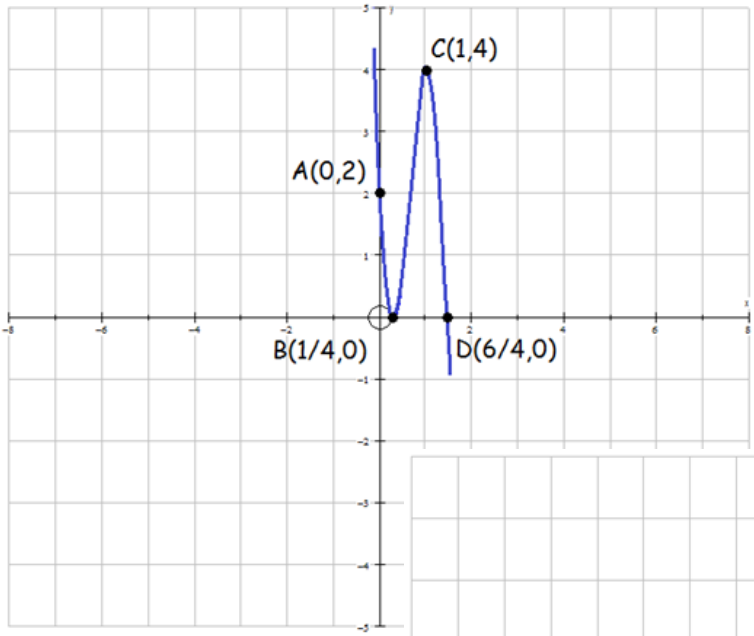
g) $f(x) - 2$ **(3)**



translation along OY
in a negative direction

moves every point down by 2 units

h) $f(4x)$ **(3)**



stretch in OX

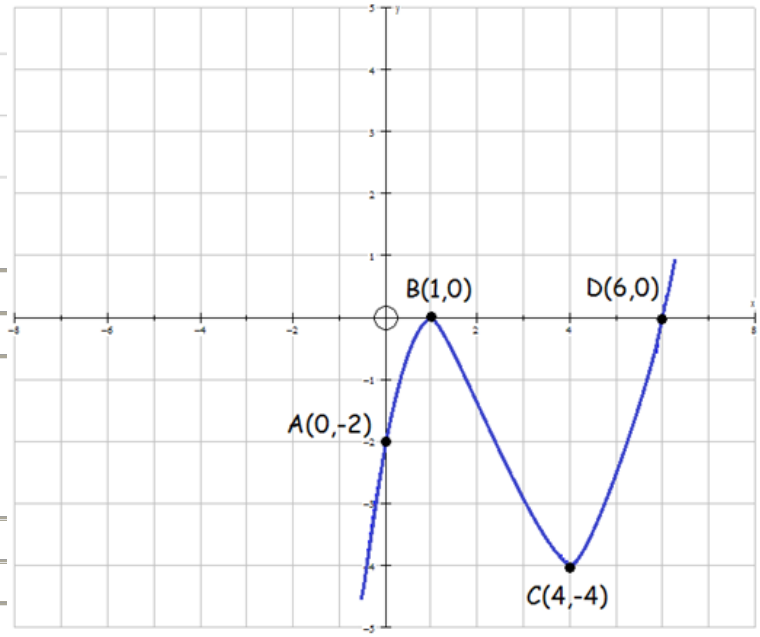
by factor of $\frac{1}{4}$

the x-coordinate

of each point is $\frac{1}{4}$

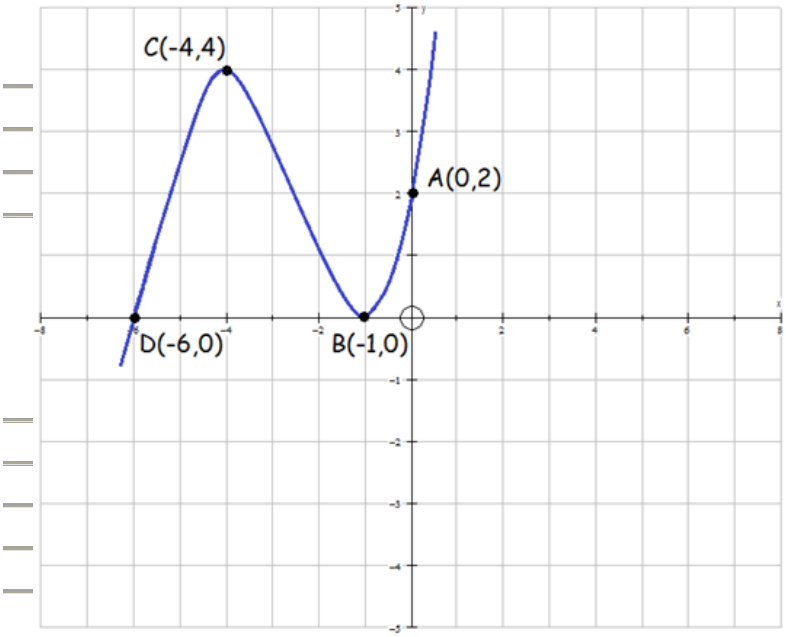
of its value

i) $-f(x)$ **(3)**



reflection in the x-axis

(x-axis is the mirror line)



j) $f(-x)$ **(3)**

reflection in the y-axis

(y-axis is the mirror line)

(Total **15** marks)