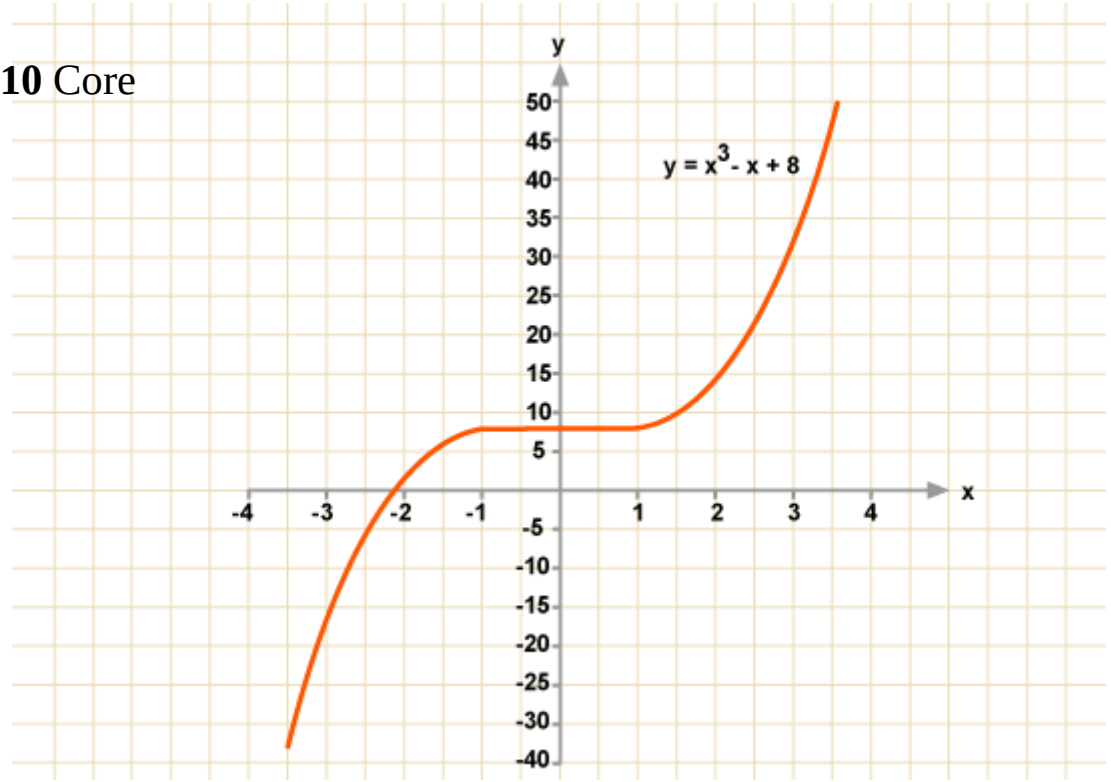
Core Mathematics

ANSWERBOOK

Unit 1

Cubic Functions and Stationary Points

Assignment **10** Core



Calculator **not** allowed (for factorising)

Calculator allowed (only for finding Stationary Points)

You must show all working

You must label all the points and graphs

Total marks for the paper – **90**

Q1

Sketch the graph of the following functions, using either *algebraic division* or the method of *comparing of the coefficients*. Use *differentiation* to find local maximum and minimum points. Show the points of intersection with the axes:

$y = x^3 - 6x^2 + 11x - 6$ **(15)**

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x = 2 is a factor $(2^3 - 6\times2^2 + 11\times2 - 6 = 0)$

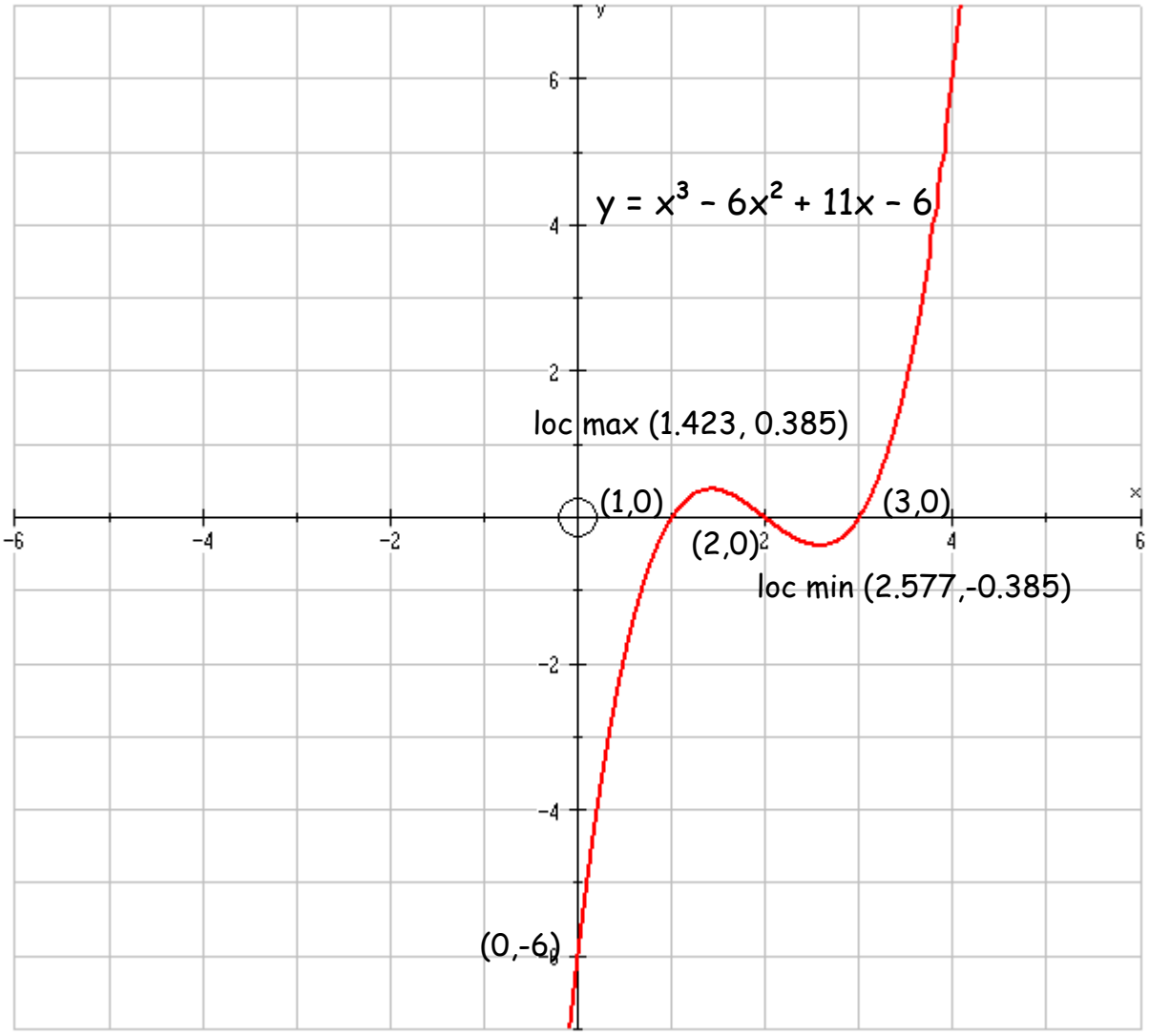
$(x^3 - 6x^2 + 11x - 6) \div (x - 2) = x^2 - 4x + 3$

$x^2 - 4x + 3 = (x - 1)(x - 3) \Rightarrow$ x = 1 or x = 3 (1,0) (2,0) (3,0)

x = 0 y = -6 (0,-6)

$\frac{dy}{dx} = 3x^2 - 12x + 11$ $3x^2 - 12x + 11 = 0$ x = 1.423 or x = 2.577

stationary points (1.423, 0.385) (2.577,-0.385)



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$y = x^3 + 2x^2 - x - 2$ **(15)**

$x = 1$ is a factor $(1^3 + 2\times1^2 - 1 - 2 = 0)$

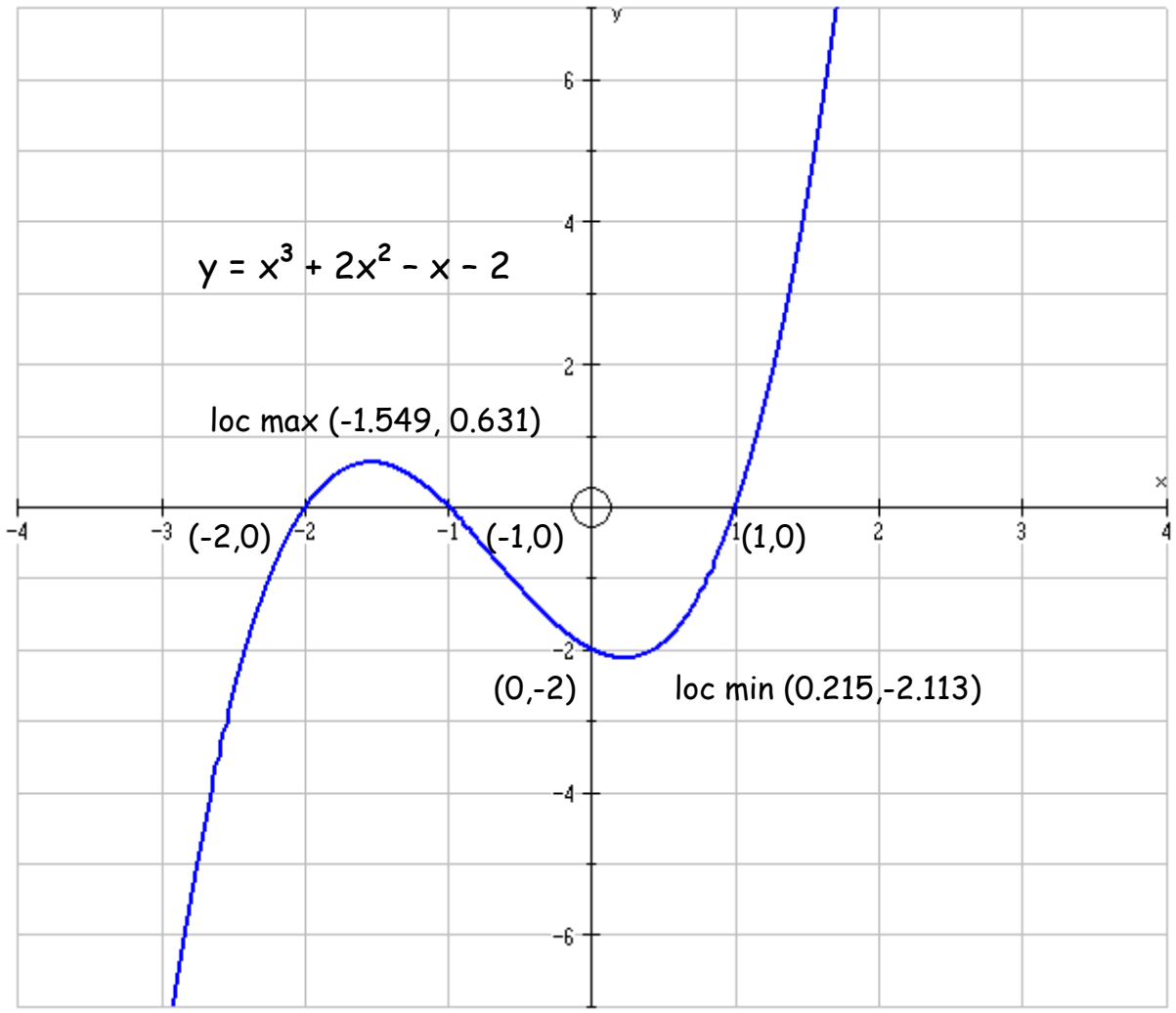
$(x^3 + 2x^2 - x - 2) \div (x - 1) = x^2 + 3x + 2$

$x^2 + 3x + 2 = (x + 2)(x + 1) \Rightarrow x = -2$ or $x = -1$ (-2,0) (-1,0) (1,0)

$x = 0$ $y = -2$ (0,-2)

$\frac{dy}{dx} = 3x^2 + 4x - 1$ $3x^2 + 4x - 1 = 0$ $x = -1.549$ or $x = 0.215$

stationary points: (-1.549, 0.631) (0.215,-2.113)



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$y = x^3 + x^2 - 4x - 4$ **(15)**

$x = 2$ is a factor $(2^3 + 2^2 - 4\times2 - 4 = 0)$

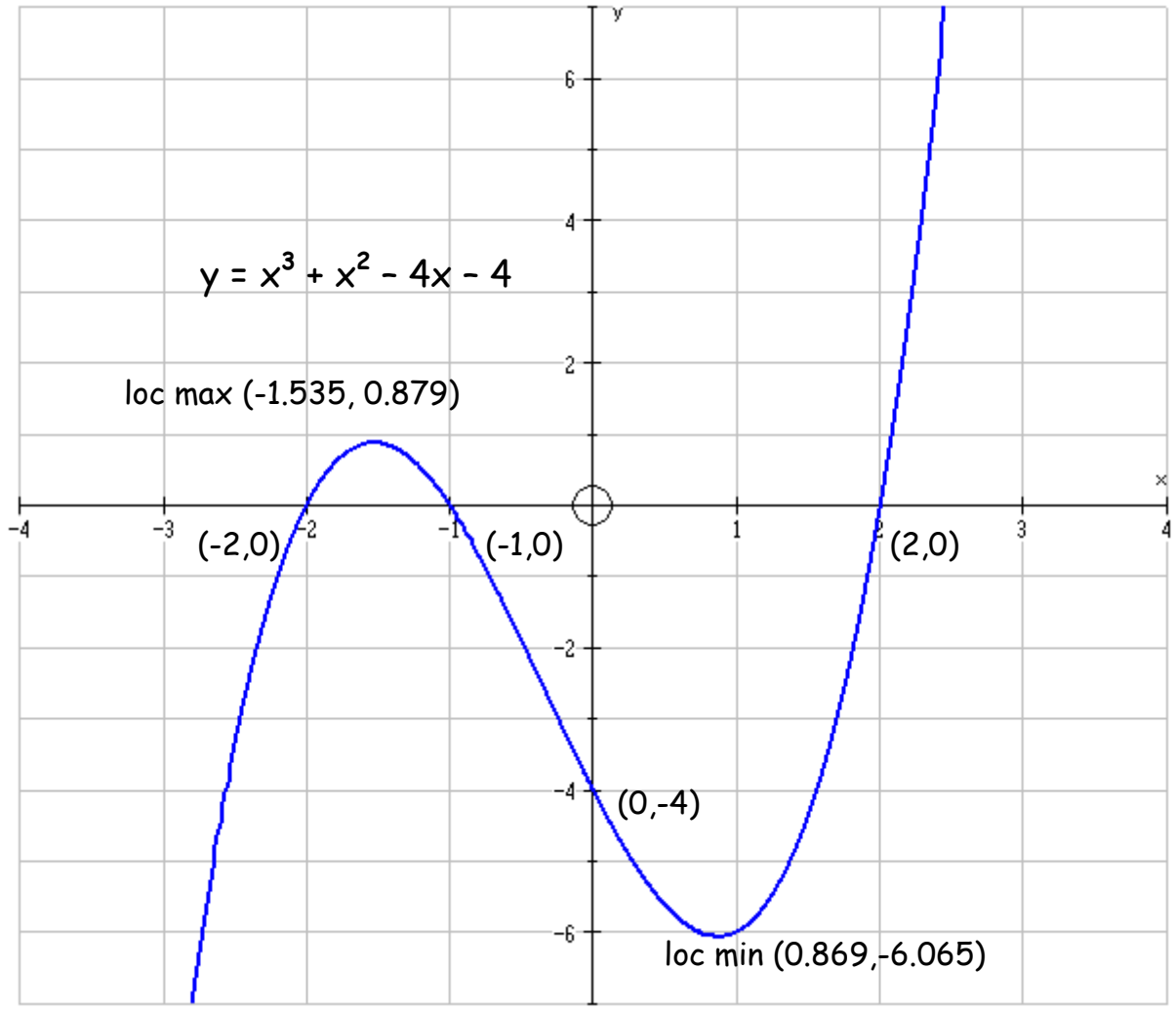
$(x^3 + x^2 - 4x - 4) \div (x - 2) = x^2 + 3x + 2$

$x^2 + 3x + 2 = (x + 2)(x + 1) \Rightarrow x = -2$ or $x = -1$ (-2,0) (-1,0) (2,0)

$x = 0$ $y = -4$ (0,-4)

$\frac{dy}{dx} = 3x^2 + 2x - 4$ $3x^2 + 2x - 4 = 0$ x = -1.535 or x = 0.869

stationary points: (-1.535, 0.879) (0.869,-6.065)



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$y = x^3 + 3x^2 - 4x - 12$ **(15)**

x = 2 is a factor $(2^3 + 3\times2^2 - 4\times2 - 12 = 0)$

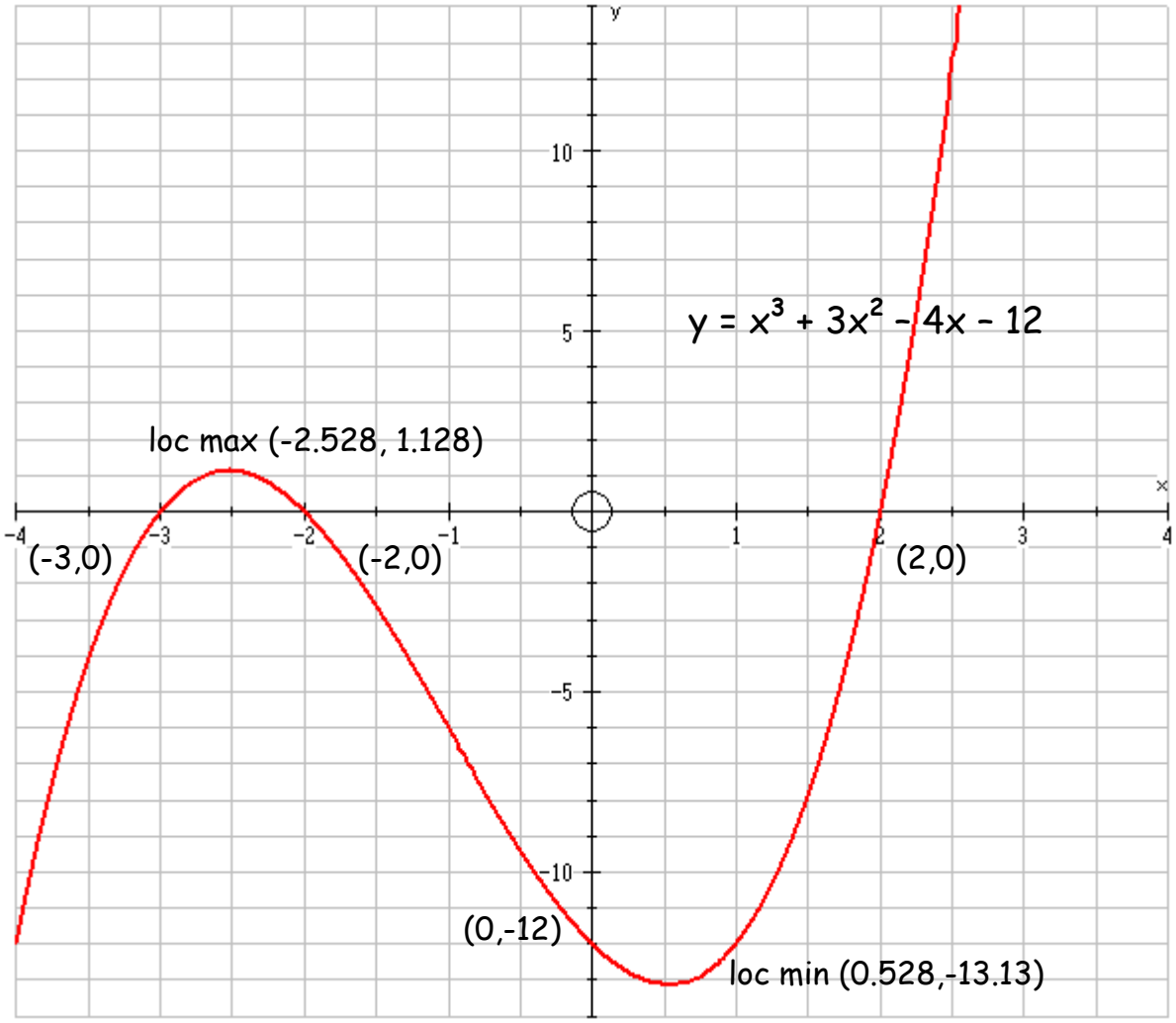
$(x^3 + 3x^2 - 4x - 12) \div (x - 2) = x^2 + 5x + 6$

$x^2 + 5x + 6 = (x + 3)(x + 2) \Rightarrow$ x = -3 or x = -2 (-3,0) (-2,0) (2,0)

x = 0 y = -12 (0,-12)

$\frac{dy}{dx} = 3x^2 + 6x - 4$ $3x^2 + 6x - 4 = 0$ x = -2.528 or x = 0.528

stationary points: (-2.528, 1.128) (0.528,-13.13)



(Total **60** marks)

Q1

Q2

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Given that $f(x) = (x^2 - 6x)(x - 2) + 3x$

a) express $f(x)$ in the form $x(ax^2 + bx + c)$, where a, b and c are constants **(3)**

$f(x) = x^3 - 6x^2 - 2x^2 + 12x + 3x$

$f(x) = x^3 - 8x^2 + 15x$

$f(x) = x\,(x^2 - 8x + 15)$

b) hence factorise $f(x)$ completely **(4)**

$f(x) = x\,(x^2 - 8x + 15)$

$f(x) = x\,(x - 3)(x - 5)$

c) sketch the graph of $y = f(x)$, showing the coordinates of each point at which the graph meets the axes. **(8)**

$f(x) = x\,(x - 3)(x - 5) \quad \Rightarrow \quad x = 0,\ x = 3 \text{ or } x = 5 \quad (0,0)\ (3,0)\ (5,0)$

$x = 0 \quad y = 0 \quad (0,0)$

$\frac{dy}{dx} = 3x^2 - 16x + 15 \qquad 3x^2 - 16x + 15 = 0 \qquad x = 1.214 \quad \text{or} \quad x = 4.12$

stationary points: (1.214, 8.209) (4.12,-4.061)

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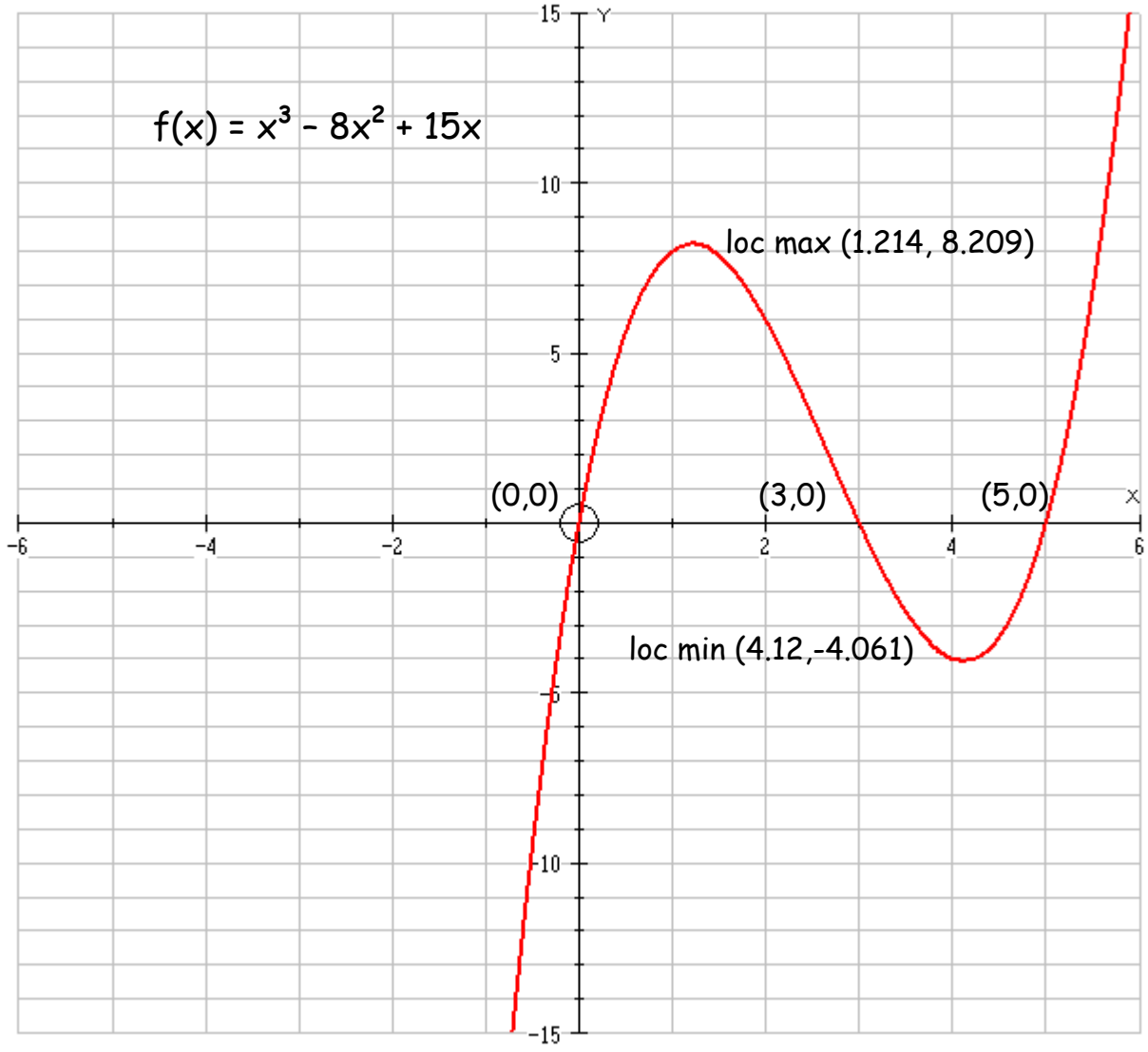

(Total **15** marks)

Q2

Q3

On the same axes sketch the graphs of the curves with equations

i $y = x^2(x-2)$

ii $y = x(6-x)$

and indicate on your sketches the coordinates of all the points where the curves cross the axes. (**10**)

i $y = x^3 - 2x^2$ $y = x^2(x-2) \Rightarrow$ $x = 0$ or $x = 2$ (0,0) (2,0)

$x = 0$ $y = 0$ (0,0)

$y' = 3x^2 - 4x$ $3x^2 - 4x = 0$ $x = 0$ or

$x = 4/3$ (0, 0) (1.333,-1.185)

ii $y = 6x - x^2$ $y = x(6-x) \Rightarrow$ $x = 0$ or $x = 6$ (0,0) (6,0)

$x = 0$ $y = 0$ (0,0)

$y' = 6 - 2x$ $6 - 2x = 0$ $x = 3$ (3, 9)

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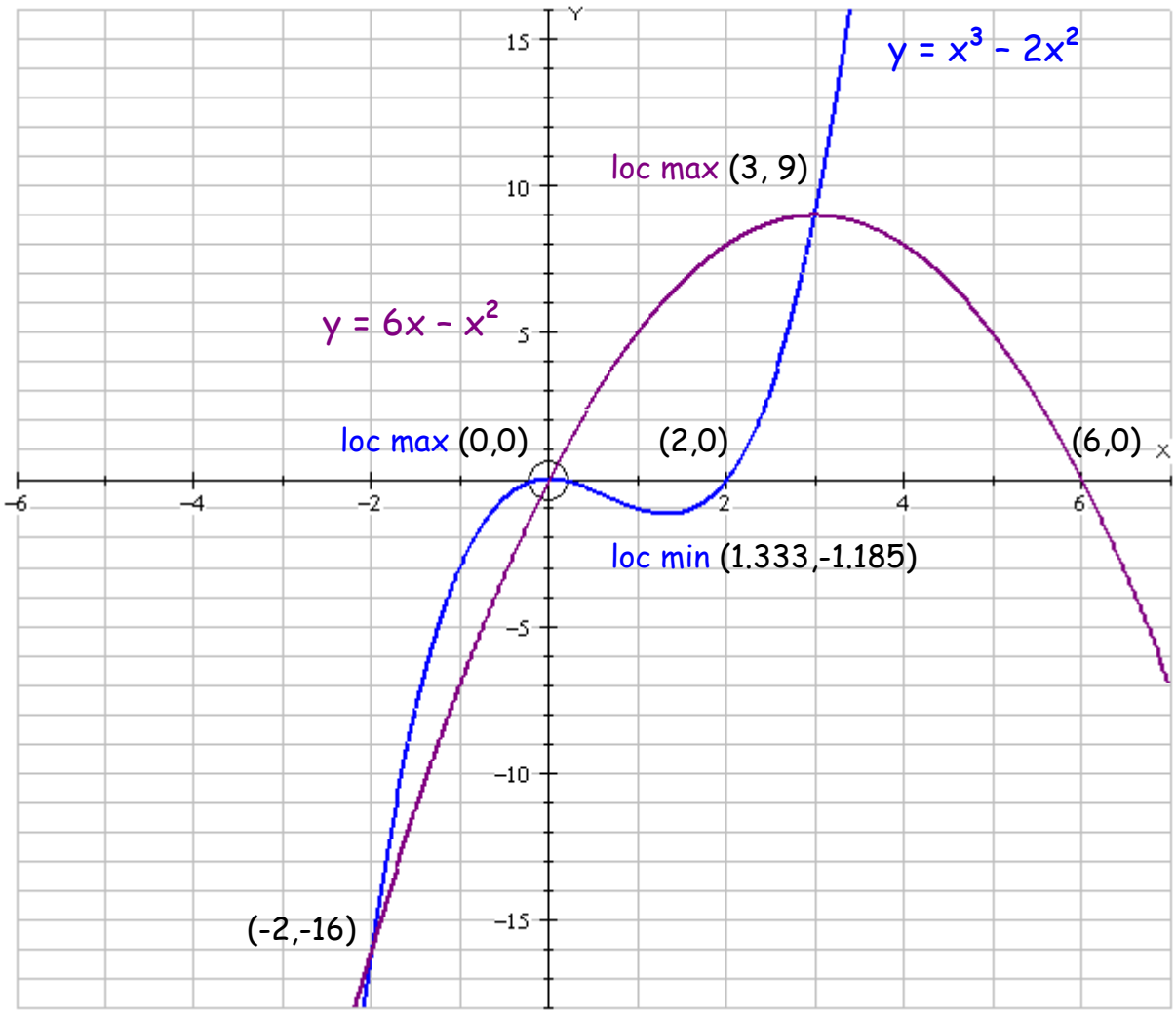

b) Use algebra to find the coordinates of the points where the graphs intersect. **(5)**

$$\begin{cases} y = x^3 - 2x^2 \\ y = 6x - x^2 \end{cases}$$

$$x^3 - 2x^2 = 6x - x^2$$
$$x^3 - x^2 - 6x = 0$$
$$x(x^2 - x - 6) = 0$$
$$x(x+2)(x-3) = 0$$

$x_1 = 0$, $x_2 = -2$ and $x_3 = 3$

$y_1 = 0$, $y_2 = -16$ and $y_3 = 9$

the points of intersection: (-2,-16) (0,0) (3,9)

Q3

(Total **15** marks)